□ Class 12 Mathematics – Chapter: Three Dimensional Geometry

---

1. Introduction

- Study of points, lines, and planes in three-dimensional space.
- Coordinates are represented as (x,y,z)(x, y, z)(x,y,z).

---

2. Distance Between Two Points

- For points P(x1,y1,z1)P(x_1, y_1, z_1)P(x1,y1,z1) and Q(x2,y2,z2)Q(x_2, y_2, z_2)Q(x2,y2,z2),

$$d=(x2−x1)2+(y2−y1)2+(z2−z1)2d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}d=(x2−x1)2+(y2−y1)2+(z2−z1)2$$

---

3. Section Formula in 3D

- Point dividing the line segment joining PPP and QQQ in ratio m:nm:nm:n is

$$(mx2+nx1m+n,my2+ny1m+n,mz2+nz1m+n)\left(\frac{m x_2 + n x_1}{m + n}, \frac{m y_2 + n y_1}{m + n}, \frac{m z_2 + n z_1}{m + n}\right)(m+nmx2+nx1,m+nmy2+ny1,m+nmz2+nz1)$$

---

4. Direction Cosines and Direction Ratios

- Direction cosines α, β, γ satisfy

$$\cos^2\alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

- Direction ratios are proportional to direction cosines.

---

5. Equation of a Line in 3D

- **Vector form:**

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

where $\vec{a}$ is position vector of a point and $\vec{b}$ is direction vector.

- **Cartesian form:**

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

---

6. Angle Between Two Lines

- If direction ratios of lines are (l_1, m_1, n_1) and (l_2, m_2, n_2), then angle θ between them is

cos⁡θ=l1l2+m1m2+n1n2l12+m12+n12l22+m22+n22\cos\theta = \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{l_1^2 + m_1^2 + n_1^2} \sqrt{l_2^2 + m_2^2 + n_2^2}}cosθ=l12+m12+n12l22+m22+n22l1l2+m1m2+n1n2

---

7. Skew Lines

- Lines that are neither parallel nor intersecting.
- Distance between skew lines formula uses vector cross product.

---

8. Equation of a Plane

- Plane passing through point (x1,y1,z1)(x_1, y_1, z_1)(x1,y1,z1) with normal vector n⃗=(A,B,C)\vec{n} = (A, B, C)n=(A,B,C) is

A(x−x1)+B(y−y1)+C(z−z1)=0A(x - x_1) + B(y - y_1) + C(z - z_1) = 0A(x−x1)+B(y−y1)+C(z−z1)=0

- General form:

Ax+By+Cz+D=0Ax + By + Cz + D = 0Ax+By+Cz+D=0

---

9. Angle Between Two Planes

-

If normals are n⃗1\vec{n}_1n1 and n⃗2\vec{n}_2n2, angle ϕ\phiϕ between planes is

cos⁡ϕ=∣n⃗1⋅n⃗2∣∣n⃗1∣∣n⃗2∣\cos\phi = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}
cosϕ=∣n1∣∣n2∣∣n1⋅n2∣

---

10. Distance from a Point to a Plane

- Distance ddd from point (x0,y0,z0)(x_0, y_0, z_0)(x0,y0,z0) to plane Ax+By+Cz+D=0Ax + By + Cz + D = 0Ax+By+Cz+D=0 is

d=∣Ax0+By0+Cz0+D∣A2+B2+C2d = \frac{|A x_0 + B y_0 + C z_0 + D|}{\sqrt{A^2 + B^2 + C^2}}
d=A2+B2+C2∣Ax0+By0+Cz0+D∣

---

11. Applications

- Geometry problems in space.
- Physics problems involving vectors and forces.
- Computer graphics and engineering.

---

12. Exam Tips

- Memorize formulas for distance and angles.

- Practice converting between vector and Cartesian forms.
- Understand geometric interpretations of lines and planes.
- Solve variety of problems on skew lines and planes.